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2005 J. Phys. A: Math. Gen. 38 L589

(http://iopscience.iop.org/0305-4470/38/36/L01)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) L589–L595 doi:10.1088/0305-4470/38/36/L01

LETTER TO THE EDITOR

No sliding in time

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Received 1 July 2005
Published 23 August 2005
Online at stacks.iop.org/JPhysA/38/L589

Abstract

In this letter, we analyse the following apparent paradox: as has been recently proved by Hastings (2004 *Phys. Rev.* **69** 104431), under a general set of conditions, if a *local* Hamiltonian has a spectral gap above its (unique) ground state (GS), all connected equal-time correlation functions of local operators decay exponentially with distance. On the other hand, statistical mechanics provides us with examples of 3D models displaying so-called sliding phases (O'Hern *et al* 1999 *Phys. Rev. Lett.* **83** 2745) which are characterized by the algebraic decay of correlations within 2D layers and exponential decay in the third direction. Interpreting this third direction as time would imply a gap in the corresponding (2+1)D quantum Hamiltonian which would seemingly contradict Hastings' theorem. The resolution of this paradox lies in the non-locality of such a quantum Hamiltonian.

PACS numbers: 05.30.−d, 73.43.Nq

1. Introduction

When discussing the properties of a quantum many-body Hamiltonian, one commonly intermixes the notions of *gaplessness* and *criticality*. These two properties, however, may or may not automatically imply each other. The former refers to the absence of an energy gap separating the GS(s) from the excited states and implies slower than exponential, typically power-law decay of correlations in imaginary time. The latter refers to the equal-time correlations of operators separated in space. Naturally, if the system is Lorentz-invariant, the two properties are identical. Even in the absence of Lorentz invariance, these two properties typically follow from each other. For example, at a quantum critical point, there is a dynamical critical exponent z relating the scaling of correlations in space and in time: $t \propto x^z$. Since all standard examples of quantum critical points are characterized by finite non-zero values of z, gaplessness once again implies criticality and vice versa. However, a spin model introduced

recently in the context of systems with topological order [3], was shown to be gapless while possessing only short-ranged equal-time correlations between *local* operators. Moreover, in that particular model one could add another term to the Hamiltonian which would open a gap without affecting equal-time correlations. This last feature, of course, should not come as a surprise since for any model with short-ranged ground-state correlations, one could just add to the Hamiltonian an operator $(\mathbb{I} - \mathcal{P}_0)$ where $\mathcal{P}_0$ is a projector onto this ground state. Such a projector will necessarily open a spectral gap while the quasi-local nature of ground-state correlations will almost certainly lead to the quasi-local nature of the contributing terms in the Hamiltonian. It is the former feature, namely the spectral gaplessness of a system with short-range correlations that comes as a surprise [5]. As discussed in [3], this is due to a 'bottle-neck' quantum dynamics that does not allow for the efficient mixing of different quantum states contributing to the ground state and thus allows one to construct a 'twisted' excited state (in the spirit of Lieb–Schultz–Mattis theorem [4, 1]) whose energy is vanishing in the thermodynamic limit. A similar situation was encountered in models [5] where the 'bottle-neck' quantum dynamics, in addition to leading to energy gaps that are exponentially small in domain sizes, prevents the systems from reaching the true ground state when coupled to a quantum bath at zero temperature, remaining in a state of 'quantum glassiness' [6].

A natural question then arises: could we also find a system described by a *local* Hamiltonian with a spectral gap which nevertheless displays power-law decay of equal-time correlations of *local* operators. The condition of locality is important here: relaxing the requirement of locality of a Hamiltonian leads to a trivial 'yes' by the means of adding a projector onto the critical ground state as described in the previous paragraph. We also know examples of gapped systems with quasi-long-ranged correlations of non-local operators such as the non-local order parameters in the quantum Hall effect [7–9] and other topological phases [10, 3]. In short, the answer to the above question is 'No' due to the theorem proved by Hastings [1]. This, however, contradicts our intuition drawn from Statistical Mechanics where *sliding* phases are known to exist, as has been demonstrated in the context of stacks of 2D layers of XY spins with gradient coupling between the layers [2]. The resulting *sliding* phase is characterized by an algebraic decay of in-plane correlations within the layer and an exponential decay in the perpendicular direction. Later, this construction has been extended to the quantum systems of stripes [11, 12], with the layers now representing 1D stripes in space-time. Such phases were also found in dissipative Josephson junction arrays [13, 14].

In general, it is common in the study of quantum critical phenomena of d-dimensional systems to relate the problem to $(d + 1)$-dimensional classical systems. This can be done generically, and perhaps one place where a true difference between quantum systems in Euclidean time and classical systems may arise is when a topological term exists and the resulting classical action cannot be made real. Typically, in going from a d-dimensional quantum system with local interactions and dynamics to a $(d + 1)$-dimensional classical system, one maps the local quantum Hamiltonian density into a local classical Lagrangian density. Hence, if the critical properties of this local classical system can be understood, so would those of the quantum model. Now, one could turn the question around, and ask instead about a behaviour of a model derived from a local classical Lagrangian. Here we would like to address this question in a particularly interesting case—that of a classical sliding phase. A quantum version of this model would imply critical correlations in space but not in time, if the direction perpendicular to the layers is taken to be Euclidean time. Thus the system

[5] Note that such a possibility is remarkable from the point of view of understanding *glasses*: one of the puzzles about a generic glassy behaviour is their slow dynamics with no long spatial correlations.

is expected to be gapped and yet have critical correlations in space, seemingly a violation of Hastings' theorem. The purpose of this Letter is to examine this apparent contradiction.

2. Sliding phase

We now quickly review the sliding phase as described in [2]. The *classical* Hamiltonian consists of three parts:

$$H = H_0 + H_g + H_J. \tag{1}$$

Here H_0 is just a sum over all layers of independent XY-Hamiltonians

$$H_0 = \frac{K}{2} \sum_n \int d^2r \, [\nabla_\perp \theta_n(\mathbf{r})]^2 \,, \tag{2}$$

where $\mathbf{r} = (x, y, 0)$ is a point in the x–y plane and $\nabla_\perp$ is the gradient operator acting on these two coordinates.

The second term in equation (1) couples gradients of θ_n in different layers:

$$H_g = \frac{1}{2} \sum_{n,m} \int d^2r \frac{U_m}{2} \{\nabla_\perp [\theta_{n+m}(\mathbf{r}) - \theta_n(\mathbf{r})]\}^2. \tag{3}$$

Finally, Josephson couplings between layers are added

$$H_J = -V_J \int d^2r \cos[\theta_{n+p}(\mathbf{r}) - \theta_n(\mathbf{r})]. \tag{4}$$

Following [2], we choose to consider only two-layer couplings with p being the distance between the coupled layers. For the reasons that will become clear shortly, we will concentrate on the next-nearest layer coupling, $p = 2$. As follows from [2], the physics of the sliding phase should not depend on this choice.

In the absence of the gradient couplings (second term in equation (1)), this Hamiltonian is just that of a 3D XY-model (strictly speaking, for $p = 2$, it is two independent interlaced XY-models). It might appear unorthodox that we keep the cosine coupling between the layers while using its expanded form for the in-plane couplings. The continuous in-plane limit is not an issue here; we simply adopt it from [2]. This choice of couplings is permitted keeping in mind the ultimate goal of constructing the phase with critical intra-layer and exponential inter-layer correlations. In such a phase, one can ignore in-plane vortices but must allow for between-the-planes ones. However, without the gradient couplings, there is no such phase as the temperature at which layers decouple turns out to be higher than the Kosterlitz–Thouless (KT) temperature in a single layer.

On the other hand, in the absence of Josephson couplings ($V_J = 0$), one obtains the *ideal* sliding Hamiltonian $H_S = H_0 + H_g$, which is invariant with respect to $\theta_n(\mathbf{r}) \to \theta_n(\mathbf{r}) + \psi_n$ for any constant ψ_n. That is, the energy is unchanged when angles in different layers slide relative to one another by arbitrary amounts—one can think of it as a 'reduced' gauge symmetry with one gauge choice per layer. As a result, the angles in different layers are uncorrelated. The low-temperature phase is the *ideal* sliding phase characterized by $\cos[\theta_m(\mathbf{r}) - \theta_n(0)] \sim \delta_{m,n} r^{-\eta}$ with $\eta = T/(2\pi\tilde{K})$, where $\tilde{K}$ is the renormalized in-plane coupling. For the simplest case of only nearest-layer coupling $U_m = U\delta_{m,\pm 1}$, the effective in-plane coupling is $\tilde{K} = K\sqrt{1 + 4U/K}$.

What constitutes the main result of [2], is the fact that in the presence of *both* the gradient and Josephson couplings between the layers, a careful choice of coupling constants may open a 'window of opportunity' in temperature within which the layers decouple before the spins in each layer completely disorder via a KT transition. In other words, the inter-layer vortices proliferate before the in-plane vortices unbind. This is the sliding phase which we shall now discuss in the quantum context.

3. Quantum Hamiltonian

Our discussion of constructing the corresponding quantum Hamiltonian will follow Kogut's review [15]. If the classical Hamiltonian H consists of only intra-layer terms, $H'[\theta_n]$ and terms coupling neighbouring layers, $H''[\theta_n, \theta_{n+1}]$, the corresponding partition function can be written as a $Z = \mathrm{tr}(\hat{T}^N)$ where the interlayer transfer matrix is defined as

$$\langle\theta_{n+1}|\hat{T}|\theta_n\rangle = \exp\left\{-\beta\left(\tfrac{1}{2}H'[\theta_n] + \tfrac{1}{2}H'[\theta_{n+1}] + H''[\theta_n, \theta_{n+1}]\right)\right\} \tag{5}$$

and N is the total number of layers. We now identify the nth layer with the imaginary time slice τ_n while $\tau_{n+1} = \tau_n + \epsilon$. The quantum Hamiltonian $\hat{\mathcal{H}}$ is then defined by $\langle\theta(\tau_n + \epsilon)|\exp\{-\epsilon\hat{\mathcal{H}}\}|\theta(\tau_n)\rangle = \langle\theta(\tau_n + \epsilon)|\hat{T}|\theta(\tau_n)\rangle$ in the limit of $\epsilon \to 0$. Note that the explicit form of a quantum Hamiltonian may be difficult to obtain: the transfer matrix needs not be in the form of an exponential of a simple operator expression. It is, however, often easy to represent it as a product of such exponentials, each corresponding to a term in the classical Hamiltonian.

3.1. Ideal sliding

Let us first look at the case of an ideal sliding Hamiltonian ($V_J = 0$). For simplicity, for now we also restrict ourselves to the nearest-neighbour coupling: $U_m = U\delta_{m,\pm1}$. Denoting $\theta = \theta_n = \theta^{(i)}(\tau)$ and $\theta' = \theta_{n+1} = \theta^{(i)}(\tau + \epsilon)$ we have

$$\langle\theta'|\hat{T}_S|\theta\rangle = \exp\left[-\frac{\beta}{4}\int \mathrm{d}^2r\,(K[\nabla\theta(\mathbf{r})]^2 + K[\nabla\theta'(\mathbf{r})]^2 + 2U\{\nabla[\theta'(\mathbf{r}) - \theta(\mathbf{r})]\}^2)\right]. \tag{6}$$

Introducing the canonically conjugate quantized fields $\hat{\theta}(\mathbf{r})$ and $\hat{\pi}(\mathbf{r})$ such that $[\hat{\pi}(\mathbf{r}), \hat{\theta}(\mathbf{r}')] = -\mathrm{i}\delta(\mathbf{r}' - \mathbf{r})$, we obtain

$$\hat{T}_S \propto \exp\left\{-\frac{\beta K}{4}\int \mathrm{d}^2r[\nabla\hat{\theta}(\mathbf{r})]^2\right\}\exp\left\{\frac{1}{4\pi\beta U}\int \mathrm{d}^2r\,\mathrm{d}^2r'\hat{\pi}(\mathbf{r})\hat{\pi}(\mathbf{r}')\ln|\mathbf{r} - \mathbf{r}'|\right\}$$
$$\times\exp\left\{-\frac{\beta K}{4}\int \mathrm{d}^2r[\nabla\hat{\theta}(\mathbf{r})]^2\right\}. \tag{7}$$

This can be explicitly verified by substituting the above expression for $\hat{T}$ into equation (6). Note that the operators in the first and the third exponent of equation (7) are diagonal in the θ representation while for the second exponential we have

$$\begin{aligned}
&\langle\theta'|\exp\left\{g\int \mathrm{d}^2r\,\mathrm{d}^2r'\hat{\pi}(\mathbf{r})\hat{\pi}(\mathbf{r}')\ln|\mathbf{r} - \mathbf{r}'|\right\}|\theta\rangle \\
&\quad= \int \mathcal{D}p\mathcal{D}p'\langle\theta'|p'\rangle\langle p'|\exp\left\{g\int \mathrm{d}^2r\,\mathrm{d}^2r'\hat{\pi}(\mathbf{r})\hat{\pi}(\mathbf{r}')\ln|\mathbf{r} - \mathbf{r}'|\right\}|p\rangle\langle p|\theta\rangle \\
&\quad= \int \mathcal{D}p\exp\left\{\int \mathrm{d}^2r\left[\mathrm{i}p(\mathbf{r})[\theta'(\mathbf{r}) - \theta(\mathbf{r})] + g\int \mathrm{d}^2r'\,p(\mathbf{r})p(\mathbf{r}')\ln|\mathbf{r} - \mathbf{r}'|\right]\right\} \\
&\quad\propto \int \mathcal{D}p_k\exp\left\{\int \frac{\mathrm{d}^2k}{(2\pi)^2}\left[\mathrm{i}p_{\mathbf{k}}(\theta'_{-\mathbf{k}} - \theta_{-\mathbf{k}}) - \frac{2\pi g}{k^2}p_{\mathbf{k}}p_{-\mathbf{k}}\right]\right\} \\
&\quad\propto \exp\left\{-\frac{1}{8\pi g}\int \frac{\mathrm{d}^2k}{(2\pi)^2}k^2(\theta'_{\mathbf{k}} - \theta_{\mathbf{k}})(\theta'_{-\mathbf{k}} - \theta_{-\mathbf{k}})\right\} \\
&\quad= \exp\left(-\frac{1}{8\pi g}\int \mathrm{d}^2r\{\nabla[\theta'(\mathbf{r}) - \theta(\mathbf{r})]\}^2\right).
\end{aligned} \tag{8}$$

Instead of writing the corresponding quantum Hamiltonian formally as $\hat{\mathcal{H}}_S = -(1/\epsilon)\ln\hat{T}_S$, we can simplify it by going to the continuous time limit $\epsilon \to 0$. We introduce the new coupling constants, $g_\theta \equiv \frac{\beta K}{2\epsilon}$ and $g_\pi \equiv \frac{1}{4\pi\beta U\epsilon}$ and require that they do not scale with ϵ, which implies the following scaling for the original couplings: $K \sim \epsilon$, $U \sim \epsilon^{-1}$. Then, up to the corrections of order ϵ,

$$\hat{\mathcal{H}}_S = g_\theta \int d^2r[\nabla\hat{\theta}(\mathbf{r})]^2 - g_\pi \int d^2r d^2r' \hat{\pi}(\mathbf{r})\hat{\pi}(\mathbf{r}')\ln|\mathbf{r}-\mathbf{r}'|. \tag{9}$$

Note that the implied scaling of K and U does not lead to any problems with tuning the parameters to the values needed to reach the desired sliding phase. This is because the *effective* coupling governing the behaviour of the classical statistical mechanical model is given by

$$\beta\tilde{K} = \beta K\sqrt{1+\frac{4U}{K}} = 2\epsilon g_\theta\sqrt{1+\frac{1}{2\pi g_\theta g_\pi \epsilon^2}} \to \sqrt{\frac{2g_\theta}{\pi g_\pi}} \quad \text{as } \epsilon \to 0. \tag{10}$$

Thus, this model has a well-defined continuous time limit.

However, the quantum version of the ideal sliding Hamiltonian contains logarithmically long-ranged interactions between momenta at different points. As will be discussed below, Hastings' theorem does not apply to such a Hamiltonian, hence no contradiction appears here. We remark on an interesting physical picture arising from the quantum Hamiltonian (9) if we think of the eigenvalues of $\hat{\pi}$ as 'charge' or 'vorticity'. Note that due to the compactness of the conjugate variable θ, the eigenvalues of $\hat{\pi}$ are quantized in integer units (we used $\hbar = 1$), this is completely analogous to the quantization of L_z in quantum mechanics. Therefore the second term in the quantum Hamiltonian (9) describes a classical 2D Coulomb gas (or a gas of vortices) with the usual logarithmic interaction. We know that the collective mode (plasmon) in such gas is gapped (unlike in the case of $1/r$ interactions). The first term has no simple classical meaning in this language; it is responsible for making the correlations quasi-long-ranged. Indeed, as follows from equation (10), unless $g_\theta > g_\theta^* \equiv 2g_\pi/\pi$, the model is in its 'high-temperature' phase with both a spectral gap and exponentially decaying correlations.

One could argue, however, that an ideal sliding phase is 'pathological' in the sense of having different time slices completely uncorrelated. In what follows we argue that considering all terms in the classical Hamiltonian (1) does not alleviate the above problem of long-range interactions while bringing new ones.

3.2. Non-ideal sliding

One apparent problem arises immediately: according to [2], in order to have a sliding phase we must have additional gradient couplings between at least next-nearest layers. Since no time derivatives of momenta are allowed to appear in a Hamiltonian, such coupling seems to have no quantum analogue. This problem is, however, easily circumvented by doubling the number of components of the field θ

$$\boldsymbol{\theta}(\mathbf{r},\tau_m) = (\theta^{(1)}(\mathbf{r},\tau_m),\theta^{(2)}(\mathbf{r},\tau_m)) \equiv (\theta_{2m}(\mathbf{r}),\theta_{2m+1}(\mathbf{r})). \tag{11}$$

A single time slice is now represented by two layers. Due to the nearest-layer interactions in the original classical Hamiltonian we have now generated interactions between the two components of our quantum field $\boldsymbol{\theta}$, but this situation is not unusual: a non-linear σ-model provides a standard example of such behaviour. It must now become clear why we have chosen $p = 2$ in equation (4): the Josephson terms now couple only identical components in the nearest time slices.

Keeping track of both components, however, brings unnecessary complications. The problem with constructing a local quantum Hamiltonian is apparent even if we forget about

the two-component nature of the field and concentrate only on a single component. In what follows, ϑ will be used to represent either of the two components, $\theta^{(1)}$ or $\theta^{(2)}$, and for simplicity we will only consider the terms in the classical Hamiltonian that do not mix them

$$\tilde{H} = \frac{1}{2}\sum_n \int \mathrm{d}^2 r (K[\nabla_\perp \theta_n(\mathbf{r})]^2 + U_2\{\nabla_\perp[\theta_{n+2}(\mathbf{r}) - \theta_n(\mathbf{r})]\}^2 - 2V_\mathrm{J}\cos[\theta_{n+2}(\mathbf{r}) - \theta_n(\mathbf{r})]). \tag{12}$$

Denoting $\vartheta = \theta_n = \theta^{(i)}(\tau)$ and $\vartheta' = \theta_{n+2} = \theta^{(i)}(\tau + \epsilon)$ we have, similarly to equation (6)

$$\langle\vartheta'|\hat{\tilde{T}}|\vartheta\rangle = \exp\left[-\frac{\beta}{4}\int \mathrm{d}^2 r (K[\nabla\vartheta(\mathbf{r})]^2 + K[\nabla\vartheta'(\mathbf{r})]^2 + 2U_2\{\nabla[\vartheta'(\mathbf{r}) - \vartheta(\mathbf{r})]\}^2 - 4V_\mathrm{J}\cos[\vartheta'(\mathbf{r}) - \vartheta(\mathbf{r})])\right]. \tag{13}$$

With the help of $\exp(a\cos\phi) = \sum_m I_{|m|}(a)\exp(\mathrm{i}ma)$, where $I_n(x)$ is a modified Bessel function, we obtain, similarly to equation (7):

$$\hat{\tilde{T}} \propto \exp\left\{-\frac{\beta K}{4}\int \mathrm{d}^2 r[\nabla\hat{\vartheta}(\mathbf{r})]^2\right\}\sum_{\{m(\mathbf{r})\}}\exp\left\{\int \mathrm{d}^2 r\,\mathrm{d}^2 r'\left(\frac{1}{4\pi\beta U_2}[\hat{\pi}(\mathbf{r}) - m(\mathbf{r})][\hat{\pi}(\mathbf{r}') - m(\mathbf{r}')]\ln|\mathbf{r} - \mathbf{r}'| + \ln I_{|m(\mathbf{r})|}(\beta V_\mathrm{J})\right)\right\}\exp\left\{-\frac{\beta K}{4}\int \mathrm{d}^2 r[\nabla\hat{\vartheta}(\mathbf{r})]^2\right\}. \tag{14}$$

While obtaining the actual quantum Hamiltonian from equation (14) is much more complicated than from equation (7), the two cases share the same important feature: a long-ranged logarithmic interaction between momenta at different locations.

4. Discussions

Let us now discuss the applicability of Hastings' theorem to the quantum Hamiltonian we have attempted to construct. In short, the theorem states that for a quantum system with a *local* Hamiltonian and a unique ground state separated from excited states by a gap, all equal-time connected correlation functions of local operators $\langle 0|\hat{A}\hat{B}|0\rangle - \langle 0|\hat{A}|0\rangle\langle 0|\hat{B}|0\rangle$ decay exponentially with distance.

Therefore this theorem is not applicable to the quantum Hamiltonians (7, 14) due to logarithmic interactions appearing there. What seems counterintuitive is that these non-local interactions originate from a perfectly local 'sliding' term in the classical action.

The resolution of the original paradox appears to be one aspect of, perhaps, a broader question: when one can go from a *local* classical Lagrangian to a *local* quantum Hamiltonian. As our example shows, this needs not always be the case.

We finally remark that locality can sometimes be restored at a cost of introducing auxiliary degrees of freedom. e.g., a 'Coulomb gas' analogy for the ideal sliding Hamiltonian (9) readily hints at such a possibility: an introduction of an auxiliary gauge field—an electromagnetic vector potential—make the theory local. However, this will not contradict Hastings' theorem, as such field will come with its own gapless mode—a photon!

Acknowledgments

The authors are grateful to S Kivelson, S Chakravarty, A Kitaev, I Dimov, X-G Wen, E Fradkin, G Refael and I Klich for useful discussions and comments. KS and CN have been supported by the ARO under grant no. W911NF-04-1-0236. CN has also been supported by

the NSF under grant no. DMR-0411800. CC has been supported by the NSF under grant no. DMR-0305482.

References

[1] Hastings M B 2004 *Phys. Rev.* B **69** 104431 (*Preprint* cond-mat/0305505)
[2] O'Hern C S, Lubensky T C and Toner J 1999 *Phys. Rev. Lett.* **83** 2745 (*Preprint* cond-mat/9904415)
[3] Freedman M, Nayak C and Shtengel K 2005 *Phys. Rev. Lett.* **94** 147205 (*Preprint* cond-mat/0408257)
[4] Lieb E, Schultz T and Mattis D 1961 *Ann. Phys.* **16** 407
[5] Castelnovo C *et al* 2004 *Preprint* cond-mat/0410562
[6] Chamon C 2005 *Phys. Rev. Lett.* **94** 040402 (*Preprint* cond-mat/0404182)
[7] Girvin S M and MacDonald A H 1987 *Phys. Rev. Lett.* **58** 1252
[8] Zhang S C, Hansson T H and Kivelson S 1989 *Phys. Rev. Lett.* **62** 82
[9] Read N 1989 *Phys. Rev. Lett.* **62** 86
[10] Freedman M *et al* 2004 *Ann. Phys.* **310** 428 (*Preprint* cond-mat/0307511)
[11] Emery V J *et al* 2000 *Phys. Rev. Lett.* **85** 2160 (*Preprint* cond-mat/0001077)
[12] Vishwanath A and Carpentier D 2001 *Phys. Rev. Lett.* **86** 676 (*Preprint* cond-mat/0003036)
[13] Tewari S, Toner J and Chakravarty S 2004 *Preprint* cond-mat/0407308
[14] Tewari S, Toner J and Chakravarty S 2005 *Preprint* cond-mat/0501219
[15] Kogut J B 1979 *Rev. Mod. Phys.* **51** 659